TRANSPORT PHENOMENA OF ICE-SLURRY IN AN ICE FORMING UNIT

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ABSTRACT

In this paper, we have developed a one-domain macroscopic model to study the transport phenomena during the flow of ice-slurry in an ice forming unit. In this model, thermosolutal convection, motion of solid phase relative to liquid, the rheological parameters of ice-slurry (viscosity variations) and drag sub-models for the solid-liquid mixture in the multiphase convecting system are taken into account. Using this model, we predict the attendant transport phenomena during generation of ice slurry in a rectangular cavity. This model is applied to aqueous ammonium chloride (NH₄Cl-69 wt%H₂O) solution. We present the results for thermal field evolution, flow dynamics, solidification behaviour and sedimentation in ice slurry. As freezing proceeds, a mushy zone forms with mobile and immobile parts. In mobile part the ice particles are free, forming the ice slurry, and in the immobile part ice particles are packed. The influence of motion of ice particles from the mobile part of the mushy zone on ice-slurry flow dynamics is highlighted.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>ρ</td>
<td>Density</td>
<td>kg/m³</td>
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<tr>
<td>µ</td>
<td>Viscosity</td>
<td>Pa.s</td>
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<tr>
<td>c_p</td>
<td>Specific heat capacity</td>
<td>J/kg.K</td>
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<tr>
<td>k</td>
<td>Thermal conductivity</td>
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<tr>
<td>βₜ</td>
<td>Thermal expansion coefficient</td>
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<tr>
<td>βₑ</td>
<td>Solutal expansion coefficient</td>
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<tr>
<td>C</td>
<td>Concentration</td>
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</tr>
<tr>
<td>D_l</td>
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<td>L</td>
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<td>g</td>
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<td>t</td>
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<tr>
<td>p</td>
<td>Pressure</td>
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Subscript

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<td>Effective</td>
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<tr>
<td>cold</td>
<td>Cold</td>
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∗ Address all correspondence to this author.
INTRODUCTION
Since the beginning of past decade, there has been a budding interest in the use of ice-slurries as a phase change material for many cooling applications due to several thermo-physical and transport advantages as well as the environmentally friendly nature of ice-slurries. Ice-slurries are superior in many aspects due to the large latent heat and surface area density of ice crystals, as well as the dynamic behavior of ice-slurry. Previous investigations dealing with the melting flow and heat transfer properties of ice-slurries have been discussed by Ayel et al. [1]. Other melting heat transfer investigations include the works of Ben-Lakhdar et al. [2] and Norgard et al. [3]. An important aspect of ice-slurry flow and heat transfer is the question of non-Newtonian behavior due to the ice crystal agglomeration and possible formation of structures as studied by Yamagishi et al. [4]. There is evidence that for flow in a tube, the heat transfer coefficient may decrease with ice concentration, possibly due to re-laminarisation effects, Knodel et al. [5].
Tanino and Kozawa [6] have reported the effect of water supply patterns on the melting of ice-slurry stored in a tank. Yamaha [7] has examined the melting process under two kinds of water supply patterns. Yanadori et al. [8] have performed experiments by melting various kinds of ice particle layers mixed with water in a tank. Okada [9] has performed a melting experiment in which a packed bed of particle ice saturated with water was melted by spraying water on top of the bed. Egolf et al. [10] have presented a physical model to calculate a thermal energy storage tank in an ice-slurry system with a peak load demand. They also introduced a new concept of ice-slurry storage without mixing. Flick et al. [11] have proposed a numerical model to predict the evolution of the ice stock and the available cooling power function of the tank geometry and the operating conditions. Their model takes into account ice settling, flow circulation, ice formation and ice melting. Guilpart et al. [12] have presented some classical control methods of ice-slurry systems typically used in industry. They demonstrated that any ice accumulation in the storage tank would not prevent the ice-slurry system from functioning safely. Jamil et al. [13] have developed a differential calorimetry scanning (DSC) technique to control the ice mass fraction inside the ice-slurry storage tank. They have found that the thermodynamic equilibrium does not occur at any point in the accumulation tank during the melting process. Niezgoda-Zelasko and Zalewski have studied momentum transfer of ice-slurry flows in tubes experimentally [14] and suggested numerical modelling [15]. This growing interest towards ice-slurries has brought a large number of researchers involved in the ice-slurry generation, rheology, flow and heat transfer research and some other aspects of this potential new secondary refrigerant.

In this study the transport phenomena of ice-slurry in an ice forming unit are studied in which the motion and the solidification of ice particles are taken into consideration. A freezing point depressant (ammonium chloride salt) is added to water to suppress the freezing point of water. This way the aqueous solution becomes a binary NH₄Cl-69 wt%H₂O solution. A macroscopic model is presented to simulate the transport phenomena in the ice-slurry produced from this binary mixture in a rectangular ice making unit. The ice slurry is produced by initiating cooling from the left side of this ice making unit. The complete set of governing equations (mass, momentum, energy and species conservation) are solved using a pressure-based finite volume method incorporating a solidification model within a one-domain continuum formulation. The influence of motion of ice particles on ice-slurry flow dynamics is discussed.

MATHEMATICAL AND NUMERICAL MODELLING
Figure 1 shows the schematic diagram of ice-slurry, generated from binary aqueous NH₄Cl-69 wt%H₂O solution, undergoing freezing by cooling from the left side of the 2D rectangular cavity. In the 2-D domain all walls except the left wall are adiabatic. The left wall is maintained at constant tempera-
TABLE 1. THERMOPHYSICAL PROPERTIES FOR NH₄Cl-H₂O SYSTEM

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Density of solid, ρₛ</td>
<td>1578.00 kg/m³</td>
</tr>
<tr>
<td>Density of liquid, ρₗ</td>
<td>1078.00 kg/m³</td>
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<tr>
<td>Viscosity, μ</td>
<td>1.3×10⁻³ Pa.s</td>
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<tr>
<td>Specific heat of solid, Cₛₚ</td>
<td>1870.00 J/kg.K</td>
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<tr>
<td>Specific heat of liquid, Cₚₗ</td>
<td>3249.00 J/kg.K</td>
</tr>
<tr>
<td>Thermal conductivity of solid, kₛ</td>
<td>2.7 W/m.K</td>
</tr>
<tr>
<td>Thermal conductivity of liquid, kₗ</td>
<td>0.468 W/m.K</td>
</tr>
<tr>
<td>Diffusion coefficient, Dₛ</td>
<td>4×10⁻⁹ m²/s</td>
</tr>
<tr>
<td>Latent Heat of fusion, L</td>
<td>31380 J/kg</td>
</tr>
<tr>
<td>Thermal expansion coefficient, βₗ</td>
<td>3.032×10⁻⁴ K⁻¹</td>
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<tr>
<td>Solutal expansion coefficient, βₗ</td>
<td>0.257</td>
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<tr>
<td>Eutectic temperature, Tₑ</td>
<td>257.75 K</td>
</tr>
<tr>
<td>Eutectic concentration, Cₑ</td>
<td>0.803</td>
</tr>
</tbody>
</table>

Boussinesq approximation is made for density variation in the liquid due to both temperature and concentration. The resulting conservation equations are:

**Continuity equation**

\[ \nabla \cdot (\rho \vec{u}) = 0 \quad (3) \]

**X - Momentum equation [17]**

\[
\begin{align*}
\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{u}) &= \nabla \cdot \left( \mu \frac{\rho}{\rho_{l}} \nabla \vec{u} \right) - \frac{\partial p}{\partial x} \\
&- \frac{\mu_{l} \rho}{K_{p_{l}}} \nabla \cdot \left( \mu_{l} \frac{\rho_{f}}{\rho_{l}} \nabla u_{s} \right) \\
&+ \nabla \cdot \left( \mu_{eff} g_{s} \nabla u_{s} \right) \\
&- \nabla \cdot \left( \frac{\rho_{f}}{f_{l}} \left( u - u_{s} \right) \left( \vec{u} - \vec{u}_{s} \right) \right) \\
&- g_{s} \left( \rho_{s} - \rho_{l} \right) g \\
&+ \rho g \left[ \beta_{T} \left( T - T_{ref} \right) + \beta_{c} \left( C_{l} - C_{ref} \right) \right]
\end{align*}
\]  

**Y - Momentum equation [17]**

\[
\begin{align*}
\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{v}) &= \nabla \cdot \left( \mu \frac{\rho}{\rho_{l}} \nabla \vec{v} \right) - \frac{\partial p}{\partial x} \\
&- \frac{\mu_{l} \rho}{K_{p_{l}}} \nabla \cdot \left( \mu_{l} \frac{\rho_{f}}{\rho_{l}} \nabla v_{s} \right) \\
&+ \nabla \cdot \left( \mu_{eff} g_{s} \nabla v_{s} \right) \\
&- \nabla \cdot \left( \frac{\rho_{f}}{f_{l}} \left( v - v_{s} \right) \left( \vec{u} - \vec{u}_{s} \right) \right) \\
&- g_{s} \left( \rho_{s} - \rho_{l} \right) g \\
&+ \rho g \left[ \beta_{T} \left( T - T_{ref} \right) + \beta_{c} \left( C_{l} - C_{ref} \right) \right]
\end{align*}
\]

**μₑffective appearing in the momentum equations is the effective viscosity as defined by Thomas [18]**

\[ \mu_{eff} = \mu_{l} \left( 1 + 2.50g_{s} + 10.05g_{s}^{2} + 0.0273e^{16.60T} \right) \]  

**Energy equation [17]**

\[ \begin{align*}
\frac{\partial \left( \rho c_{p} T \right)}{\partial t} &+ \nabla \cdot \left( \rho c_{p} \vec{u} T \right) = \\
&\nabla \cdot \left( k \nabla T \right) + \left[ \frac{\partial p_{f} L}{\partial t} + \nabla \cdot \left( \rho_{f} \vec{u} L \right) \right] \\
&- \nabla \cdot \left[ \rho_{f} \left( c_{pl} - c_{ps} \right) \left( \vec{u} - \vec{u}_{s} \right) T \right]
\end{align*} \]

**Species conservation equation [17]**

At the scale of a control volume, Scheil’s assumption [19] for

**Governing Equations**

Considering a one-domain continuum model followed from Kumar et al. [17] the continuum relations are defined as

\[
\begin{align*}
g_{s} + g_{l} &= 1, \quad f_{s} + f_{l} = 1, \quad f_{l} = \frac{g_{l} \rho_{l}}{\rho}, \quad f_{s} = \frac{g_{s} \rho_{s}}{\rho}, \\
\rho &= g_{l} \rho_{l} + g_{s} \rho_{s}, \quad \vec{u} = f_{l} \vec{u}_{l} + f_{s} \vec{u}_{s}, \quad k = g_{l} k_{l} + g_{s} k_{s}, \quad D = f_{l} D_{l} + f_{s} D_{s}, \quad c_{p} = f_{l} c_{pl} + f_{s} c_{ps}
\end{align*}
\]

where \( f \) is the mass fraction, \( g \) is the volume fraction, \( k \) is the thermal conductivity, \( D \) is the diffusion coefficient, and \( c_{p} \) is the specific heat. Accordingly the continuum one-domain conservation equations for the transport variables can be written after summing over all the phases (liquid and solid phases).
microsegregation has been made where the liquid is assumed to be solutally well mixed and there is no diffusion in the solid. Also, the interface between solid and liquid is assumed to be in local thermodynamic equilibrium.

\[
\frac{\partial (\rho C_l)}{\partial t} + \nabla \cdot (\rho u C_l) = \nabla \cdot (D^+ \nabla C_l) + S_C \tag{8}
\]

where

\[
D^+ = \rho f_s D_l
\tag{9}
\]

\[
S_C = \frac{\partial \rho f_s C_l}{\partial t} - \frac{\partial \rho f_s \bar{C}_l}{\partial t} + \nabla \cdot (\rho f_s \dot{u}_l (C_l - \bar{C}_s)) \tag{10}
\]

In the present model solid volume fraction \(g_s = 0.25\) has been assumed as ice particles packing fraction. Above this fraction ice particles are packed and do not move. If solid volume fraction in a cell is below \(g_{s,cr} (0.25)\) then the viscosity submodel is used and once it becomes above \(g_{s,cr}\) then the Darcy term is activated. By this way the flow smoothly transitions from the mobile solid phase to the immobile solid phase within the mushy zone.

The solid-liquid relative velocity is given by \[20\]

\[
\dot{u}_s - \dot{u}_l = \frac{1 - g_s}{18 \mu_{eff}} (\dot{p}_s - \dot{p}_l) d_{char} 2 g_r \tag{11}
\]

where \(d_{char}\) is the characteristic size of the ice particles moving in the mobile part of the mushy zone.

Using the continuum definition for mixture velocity and the expression for relative velocity from Eq. \[11\], the solid phase velocity can be calculated as

\[
\ddot{u}_s = \ddot{u} + f_s (\dot{u}_s - \dot{u}_l) \tag{12}
\]

**NUMERICAL PROCEDURE**

In the present one-domain continuum formulation, conservation equations are employed for the entire domain, such that they are equally valid in the solid, in the liquid and the mushy regions. Using a one-domain approach, the interface is part of the solution and does not need to be tracked separately.

To test the mesh dependency of the solution, the flow of the ice-slurry in a computational domain comprising a 5cm×10cm rectangular cell (schematic of which is shown in Fig. 2), is simulated with five different grid sizes - 61×121, 71×141, 81×161,
91 × 181 and 101 × 201. The center line temperature at t = 4400 s are plotted for these grid sizes as depicted in Fig. 3. It is observed that there is negligible difference between the last two grid sizes. Hence we have discretized the computational domain in our study with 91 × 181 grids in order to save computational time. It also ensures sufficient resolution for the problem under consideration and further grid refinement doesn’t change the results appreciably. A time step of 0.1 s is chosen. Convergence in the inner iterations is declared on the basis of relative error of scalar variables to be solved (a tolerance of 10^{-4} is prescribed), as well as on satisfaction of the overall energy balance criteria within a permissible limit of 0.1%.

RESULTS AND DISCUSSION

The temperature distribution in the cavity during the cooling process is shown in Fig. 4. It is observed that due to cooling the maximum temperature in the cavity falls with time. The thermal gradient changes along with the evolution of ice slurry during solidification as shown in Fig. 5. The contour of ice packing fraction (f_s = 0.25) is superimposed on the solid fraction plot. It demarcates the freely moving ice particles in the mobile part of the mushy zone and the immobile part of the mushy zone. As ice particles are heavier, they tend to sink. They settle towards the bottom of the cavity creating a gentle ramp in the shape of the mushy zone and accumulates like a slide towards the right side of the cavity. The settling nature of solid ice particles can also be seen from the width of the non-coherent mobile zone of the mushy region (bounded by fraction solid 0-0.25). At high solid fractions ice particles consolidate and become immobile. With progress of solidification, there is an increase in the settling of solid phase. After filling the bottom of the cavity, the mobile non-coherent zone moves upward. After t = 3000s or so the cooling in the present system can be stopped to supply the ice slurry generated on demand to some cooling load system. However, in this work we studied the complete solidification of aqua-ammonia solution to understand how the ice fraction evolve in the ice-slurry at different stages of solidification.

The flow field inside the cavity at different times are depicted by Fig. 6. For clarity velocity distribution is shown by skipping every 6 grid points. Since the ice particles occupy the cavity from the start the flow pattern is very complex. The flow is typically
FIGURE 6. VELOCITY FIELDS IN THE CA VITY AT VARIOUS TIME

more pronounced in the mobile part of the mushy zone. Whereas in the immobile part the flow is typically low. We observe several clockwise and counter-clockwise rotating vortices (see velocity pattern at $t = 2000s$) as sedimentation, thermal buoyancy and solutal buoyancy, all are contributing to the flow. However after $t = 4000s$ we observe a counter-clockwise vortex representing the sedimentation of ice particles at the bottom of the cavity. This counter-clockwise vortex increases in size (see velocity pattern at $t = 6000s$) with progress in solidification. This indicates that at later time the flow is a counter-clockwise vortex mainly governed by the sedimentation of ice particles. Also it has been observed that the magnitude of maximum velocity in the cavity decreases with time. This is because of increasing amount of solid grains in the flow which increases the viscosity and hence decreases the velocity.

The evolution of ice fraction, estimated by average solid ice fraction in the domain, is shown in Fig. 7. Upto 2300s the aqua ammonia alloy is solidifying and the solute water is getting rejected into the liquid making it rich in solute water content. After 2300s we observed that the enriched water concentration reaches towards the eutectic composition (0.803 wt fraction $H_2O$) and thereafter the liquid solidifies as eutectic. Since the solidification of a liquid as an alloy and as an eutectic is different, we observe a sharp change in slope in Fig. 7 after 2300s.

CONCLUSIONS

This paper presents a numerical investigation of solidification of ice-slurry system. A new model for transport phenomena in ice slurry is developed accounting its formation in an ice making unit considering motion of ice particles and their solidification. It is observed that when significant density differences exist between precipitating solids and the liquid, as is in an $NH_4Cl$-69 wt% $H_2O$ alloy, modeling motion of ice particles is critical to successfully predicting the sedimentation behaviour and distribution of ice particles in the cavity. The main conclusions are as follows:

1. Initially the ice fraction occupies the entire cavity and tends to settle in the bottom as time passes.
2. The thermal gradient changes along with the evolution of ice slurry.
3. The flow field is observed to be very complex comprising several clockwise and counter-clockwise currents.
4. After 2300s we observed that the enriched water concentration reaches towards the eutectic composition.
5. In the present study we observed the ice particle distribution in the ice forming unit at various stages of solidification. For instance at 2000s, ice slurry with average ice fraction 7% can be supplied on demand. Similarly at 4000s, ice slurry with
15% ice fraction can be supplied on demand.

REFERENCES


